**BEFORE THE**
**FEDERAL COMMUNICATIONS COMMISSION**
**WASHINGTON, DC 20554**

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| In the Matter of | ) | |
| | ) | |
| Request for Waiver of Measurement | ) | ET Docket No. 04-352 |
| Procedures for OFDM Ultrawideband Devices | ) | |

# COMMENTS OF PHILIPS ELECTRONICS NORTH AMERICA CORPORATION


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## TABLE OF CONTENTS

**PAGE**

## EXECUTIVE SUMMARY

MBOA-SIG requests waiver of certain ultra-wideband (“UWB”) measurement procedures as they may apply to multi-band orthogonal frequency division multiplexing (MB-OFDM) systems. Philips strongly endorses grant of the Petition, and in these Comments submits additional technical data demonstrating that compared with impulse technology approved under Part 15 of the Commission’s Rules, MB-OFDM technology will not create any additional potential for interference to licensed operations.

Amplitude Probability Distribution (“APD”) plots provide helpful insight into the potential for interference to receivers without being specific to any particular receiver. In essence, an APD is the Complementary Cumulative Distribution Function (“CCDF”) of the signal amplitude expressed in dB. The National Telecommunications and Information Administration (“NTIA”) has used APDs to characterize the potential for interference to receivers, and so in these Comments we use APDs and their associated plots to describe the statistics of signal/interference waveforms.

In the first part of these Comments we analyze MB-OFDM under a variety of conditions and compare the APD plots for a simulated BPSK impulse radio. MB-OFDM employing a sequence of 3 bands creates less potential for interference than the impulse transmitters anticipated by the UWB rules. APD analysis shows that impulse radios certifiable under Part 15 may require higher SIR ratios than the proposed MB-OFDM waveform for equivalent protection of a wideband receiver. Additional data demonstrate that MB-OFDM has peak power equivalent to that of an impulse radio of equivalent PRF when measured in low bandwidth receivers, and a significantly lower peak power when measured in high bandwidth receivers.

In the second part of these Comments we demonstrate the impact of MB-OFDM on a high bandwidth QPSK transmission system. Favorable results were obtained for simulations of QPSK transmission systems using a conservative link margin.

Philips requests that the Commission consider these data and based upon this information, as well as that submitted by others, grant the waiver requested in this proceeding.

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## COMMENTS OF PHILIPS ELECTRONICS NORTH AMERICA CORPORATION

Philips Electronics North America Corporation ("Philips") respectfully submits these Comments in response to the Petition for Waiver filed by the Multi-band OFDM Alliance Special Interest Group ("MBOA-SIG") for waiver in the above referenced proceeding.[1] In its Petition, MBOA-SIG requests waiver of certain ultra-wideband ("UWB") measurement procedures as they may apply to multi-band orthogonal frequency division multiplexing ("MB-OFDM") systems. Philips is a member of the MBOA-SIG. We strongly endorse grant of the Petition, and below submit additional supporting technical data. Grant of this request is in the public interest because it will permit MB-OFDM technology to compete in the UWB marketplace without creating additional interference to licensed operations.

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[1] *See* FCC Public Notice, *Office of Engineering and Technology Declares MBOA-SIG Request for a Waiver of Part 15 for an Ultra-Wideband System to be a "Permit-but-Disclose" Proceeding for Ex Parte Purposes*, DA 04-2793 (Aug. 30, 2004), *Erratum* (Sept. 3, 2004).

**BACKGROUND**

The technical data presented below consists of two parts. The first part concentrates on analyzing MB-OFDM under a variety of conditions and compares the amplitude probability distribution ("APD") plots for a simulated BPSK impulse radio. The second part demonstrates the impact of MB-OFDM on a high bandwidth QPSK transmission system.

The APD analysis presented in Part 1 shows that UWB impulse radios certifiable under Part 15 may require higher SIR ratios than the proposed MB-OFDM waveform for equivalent protection of a wideband victim receiver (see Figure 9). This demonstrates that MB-OFDM employing a sequence of 3 bands creates less interference than the impulse transmitters anticipated by the UWB rules.

Additional data demonstrate that MB-OFDM has peak power equivalent to that of an impulse radio of equivalent PRF[2] when measured in low bandwidth receivers, and a significantly lower peak power when measured by high bandwidth receivers (see Figure 8). Favorable data also was obtained for simulations of QPSK transmission systems under the condition of a constant Eb/No of 10dB, chosen to provide a conservative link margin (see Figure 15).

Philips requests that the Commission consider these data and based upon this information, as well as that submitted by others, grant the waiver requested in this proceeding.

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[2] MB-OFDM has an equivalent PRF of $528/(165 \times 3) = 1.0\dot{6}\,\mathrm{MHz}$ for two of the four available TFI codes. The other two TFI codes have exactly half this PRF value.

## PART 1: AMPLITUDE PROBABILITY DISTRIBUTIONS

Amplitude Probability Distributions (APDs) and their associated plots can be used to succinctly describe statistics of signal/interference waveforms. The National Telecommunications and Information Administration ("NTIA") has used APDs extensively to characterize the interference potential of UWB waveforms on potential victim receivers. This is well justified, since APD plots provide helpful insight into the potential impact on victim receivers without being specific to any particular victim receiver. (If the protection of a particular communications system is to be evaluated, details of the victim system's modulation scheme and FEC performance etc., must be taken into account.)

In essence, the APD is simply the Complementary Cumulative Distribution Function ("CCDF") of the signal amplitude expressed in dB with respect to some reference. Typically, the choice of reference is either the mean power of the signal under test, or the noise floor of the measurement system.[3] By convention, the APD plot is displayed on Rayleigh graph paper, with the CCDF plotted as the abscissa and the amplitude in dB as ordinate. A similar scaling can be achieved by plotting the natural logarithm of the CCDF using a log scale for the x-axis. This scaling is equivalent to $\log_{10}(-\ln(P(A>\text{Ordinate})))$, and has been applied to the plots in this document.

The example APD plot in Figure 1 was obtained by simulation for zero-mean, complex Gaussian noise of unit power. Due to the scaling employed, the exponential function exp($x$) must be used to recover the actual probabilities. Hence, the probability the signal exceeds its mean value can be read as $\exp(-10^0) = \exp(-1) = 36.78\%$. Similarly,

[3] See http://www.its.bldrdoc.gov/home/programs/uwb_interference/tasks/EstAndGraphAPDs.pdf, page 2.

the probability that the signal at any given instant is 10 dB above its mean value can be read as $\exp(-10^1) = \exp(-10) = 0.00454\%$.

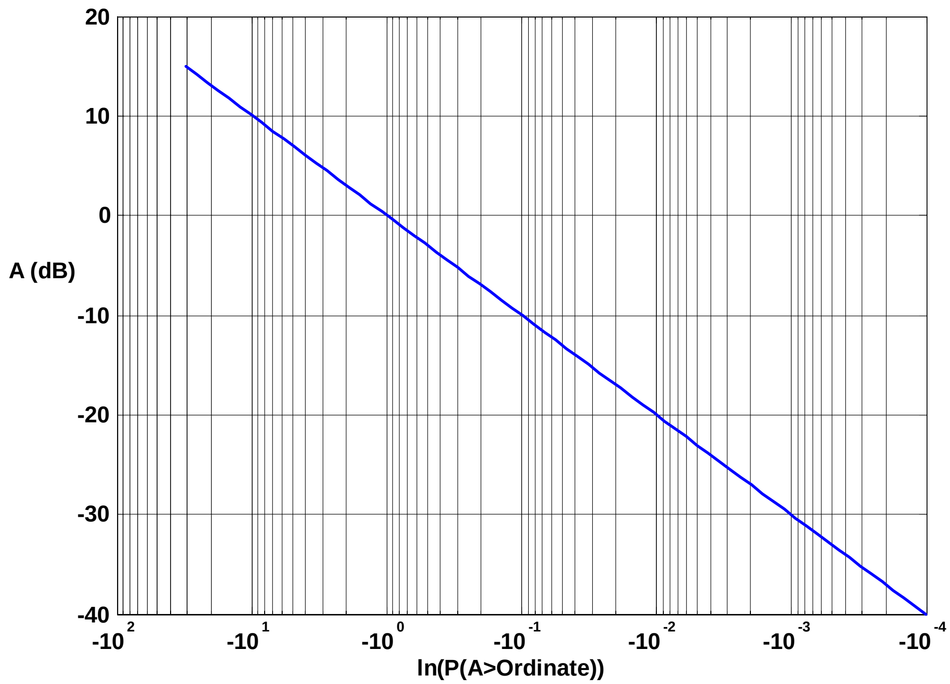


**Figure 1: Example APD for Complex Gaussian Noise Source**

The straight-line graph of the APD plot of a complex Gaussian source is due to its amplitude being Rayleigh distributed. Other signals have different characteristic shapes when plotted using this methodology, as shown in the following pages.

## APD Characterization of Continuous OFDM Signals

We examine an OFDM signal of which each sub-carrier consists of a stream of randomly chosen, equiprobable QPSK symbols because this is a good representation of the proposed MB-OFDM waveform. (We will examine the frequency agile nature of the waveform in a later section.) Since each quadrature component of the OFDM sub-carriers follows a binominal distribution, the usual approximation of the binomial by a

Normal or Gaussian distribution applies for a large number of sub-carriers. Therefore, since each quadrature component is approximately Gaussian distributed, the signal amplitude is approximately Rayleigh distributed. The discrete nature of the distribution is quite apparent with a small number of sub-carriers (see the blue line in Figure 2). However, at 8 or more sub-carriers the approximation becomes quite clear.

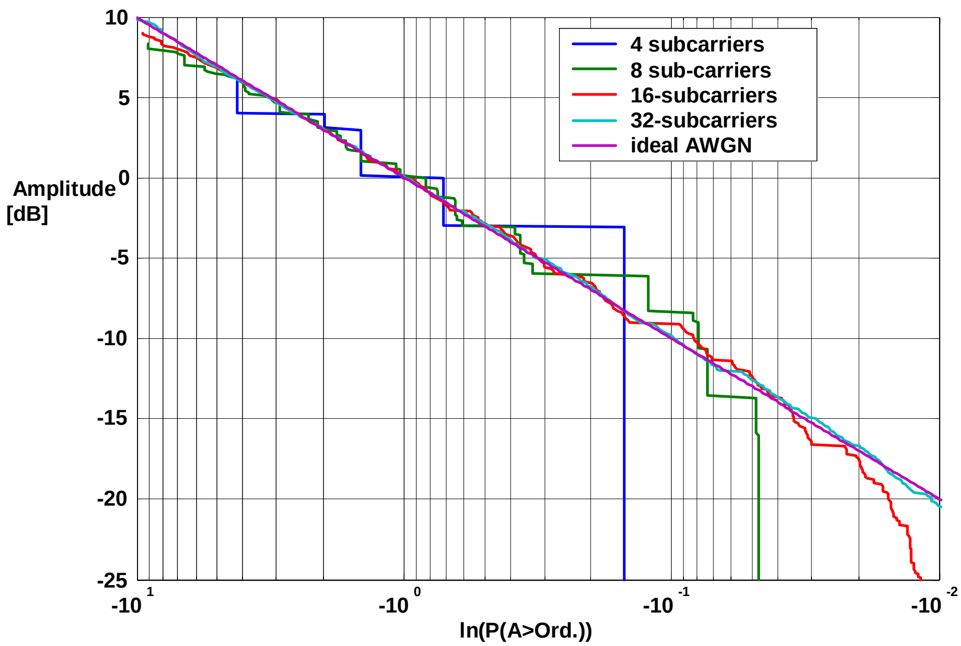


**Figure 2: APD of continuous QPSK/OFDM signals (unfiltered)**

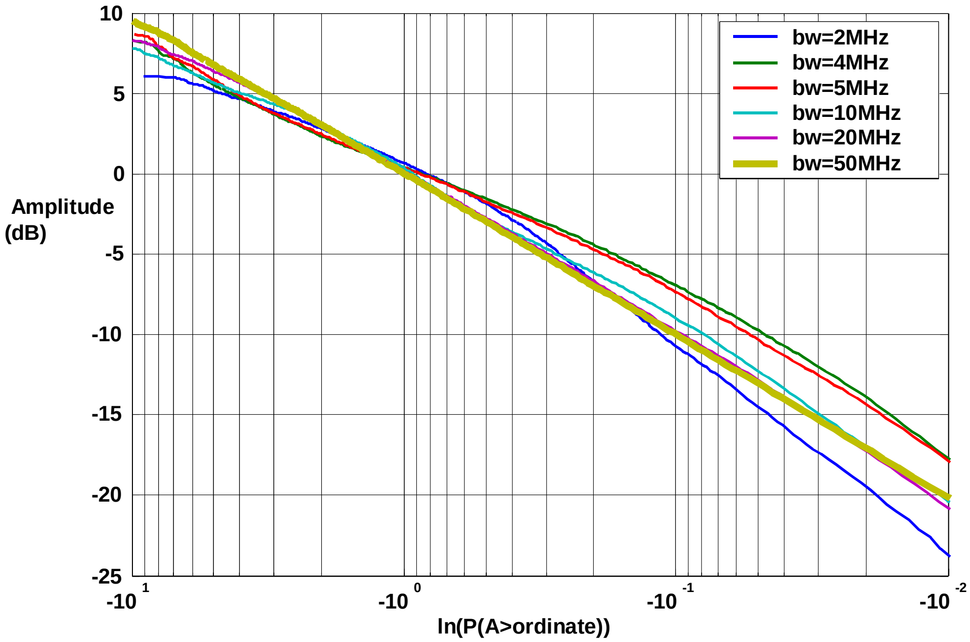


**Figure 3: APD of continuous 128-carrier QPSK/OFDM signals (filtered with various bandwidths)**

The same trend can be expected as a victim receiver's bandwidth is increased. The approximation is good for receiver bandwidths of 20 MHz or more, as may be observed from Figure 3.

### Theoretical Expectations for the APD of MB-OFDM

For measurement bandwidths of $\geq 20$ MHz, the APD of OFDM closely approximates that of a Rayleigh distribution. This can be expected because the in-phase and quadrature components both tend towards a Gaussian distribution due to the central limit theorem. Assuming this approximation to be perfect, we can write a closed form expression for the APD of OFDM and MB-OFDM (for derivations, please refer to the appendix). The wideband amplitude probability distribution as a function of the envelope amplitude $r$, is given by:

$$APD(r) = \exp(-r^2)\,.$$

When the same wideband OFDM is subject to a duty cycle, $d$ , in a particular band of interest, the amplitude probability distribution becomes modified to:

$$APD'(r) = \frac{1}{d}\exp(-r^2/d)$$

For the proposed MB-OFDM waveform the overall duty cycle is given by:

$$d = 3 \times 165/128 .$$

This calculation is based on the assumption that for any one slice of spectrum, energy from the MB-OFDM waveform is expected to be present only $1/3$ of the time, and that for any single OFDM symbol of 165 samples duration, only 128 of these samples are energized (the other samples are set to zero energy). However, it is important to consider that the equations above provide good approximations to the APD <u>only for cases of wide-band (>20 MHz) receivers where thermal noise and other interference sources are negligible</u>. Most APD curves from practical measurements will include a component of thermal noise. Furthermore, it is likely that thermal noise will dominate the interference in nearly all practical situations involving interference to wideband satellite receivers.[4]

In view of this, it is valuable to calculate APD curves for finite I/N ratios. To that end a closed form expression was derived for that case.

$$APD''(r) = \frac{(d-1)}{d}\exp\left(\frac{-r^2}{2\sigma_n^2}\right) + \frac{1}{d}\exp\left(\frac{-r^2}{2d\sigma_i^2 + 2\sigma_n^2}\right),$$

where $\sigma_n^2$ is the variance of the noise (per quadrature dimension), and $\sigma_i^2$ is the corresponding long term variance of the MB-OFDM interference. To fulfill the usual

[4] See XSI filing on ET docket 02-48, "Opposition of Xtreme Spectrum, Inc. to Petition for Reconsideration of the Satellite Industry Association"

convention for APD plots of unit mean power, we apply the constraint $2\left(\sigma_n^2 + \sigma_i^2\right) = 1$, and set the ratio $\sigma_i^2 / \sigma_n^2$ according to the desired I/N ratio.

## APD Curves for MB-OFDM

Given existence of a closed form expression for the APD curve for MB-OFDM, it is a simple matter to graph the equation, as shown in Figure 4.

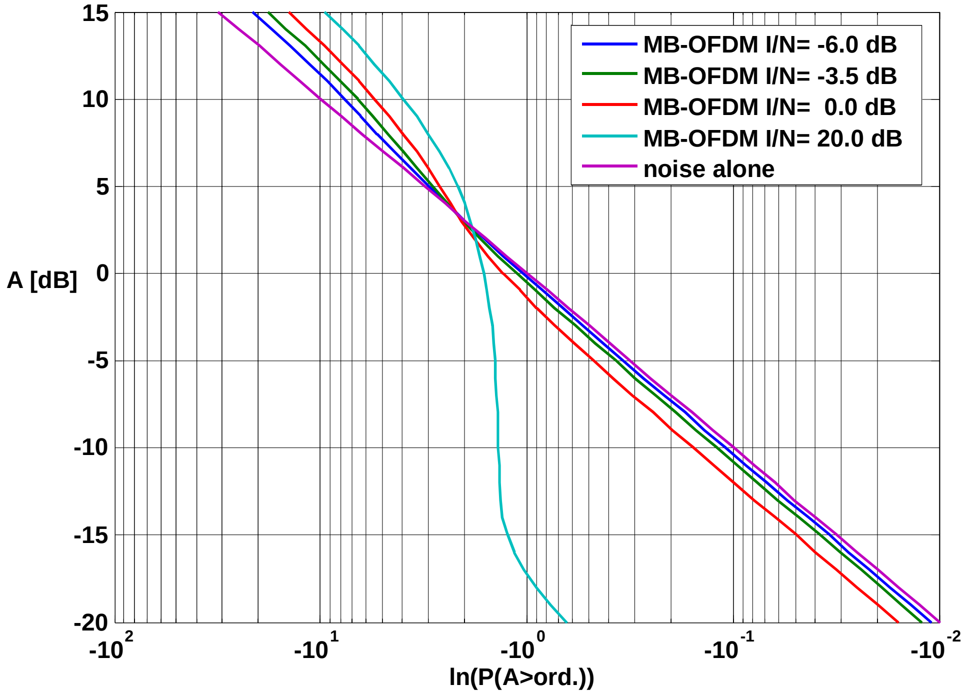


**Figure 4: Analytic APD plots for MB-OFDM in a wide bandwidth**[5]

To validate the analytic model for the APD plot, Monte-Carlo simulations were carried out to evaluate the impact of 3-band hopping as experienced by a victim receiver in one of the occupied sub-bands. Randomly modulated QPSK symbols were used and the OFDM symbols were padded with a zero energy prefix as specified in the proposed standard.

[5] We emphasize that the case of I/N=20 dB is very unlikely to occur in the field.

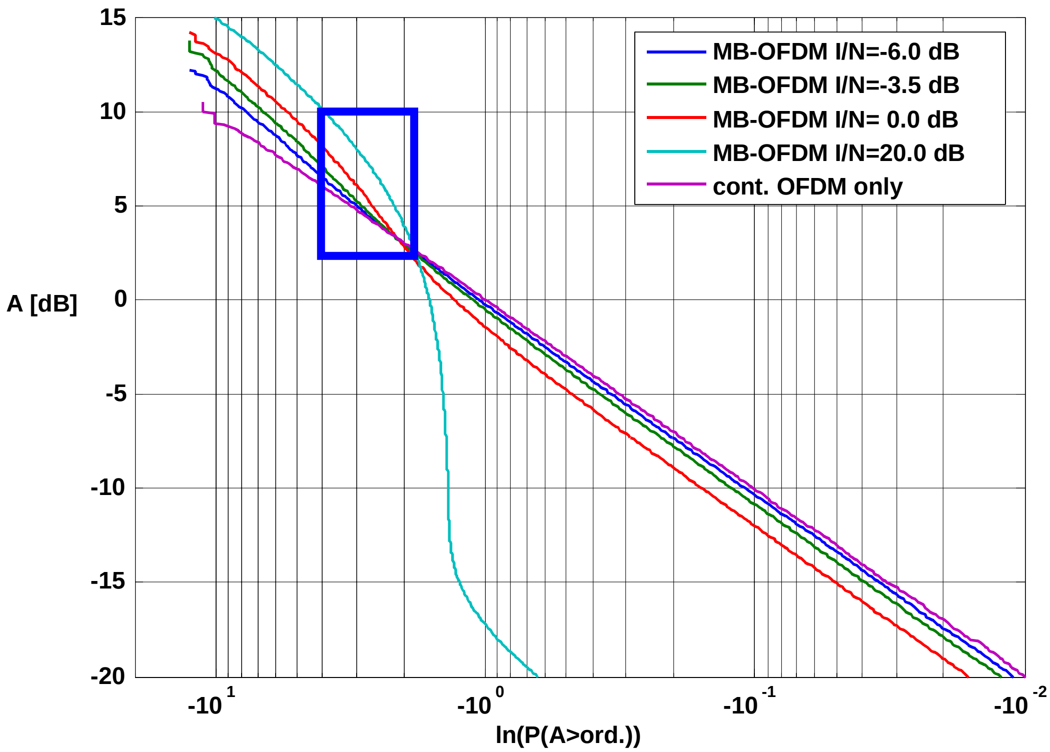


**Figure 5: Simulated APD plots for MB-OFDM and continuous OFDM in a wide bandwidth**

Good agreement is observed between the Monte-Carlo simulated APD plots of Figure 5 and the analytically derived APD plots of Figure 4. It is usual when interpreting these graphs to focus on regions where the APD plot exceeds the straight-line plot of the ideal Rayleigh distributed envelope. In the above graphs this applies for probabilities less than $\exp(-2) = 13\%$. In addition, for a digital receiver employing competent FEC techniques, amplitude peaks occurring with a probability of less than 2% can be ignored under the assumption that such low frequency error events will be corrected. In such cases, the area of interest may be limited to the rectangle indicated in Figure 5. In general, however, the degree to which the APD of MB-OFDM exceeds the AWGN case is highly dependent on the assumed I/N ratio. We again emphasize that situations in which the I/N ratio is a high positive number correspond to rare and improbable scenarios. The curve for I/N=20 dB is plotted for completeness only, and to enable comparison with results obtained in lab measurements using conducted coupling of measuring instruments.

## APD Curves for Lower Victim Receiver Bandwidths

As shown in Figure 6, Victim Rx bandwidth has a significant impact on the APD plots. Generally, lower receiver bandwidths "experience" a more benign version of the APD. Note in particular that receiver bandwidths from 2-5 MHz scarcely rise above the ideal straight-line curve for a Rayleigh distributed signal (shown as a thick black line).

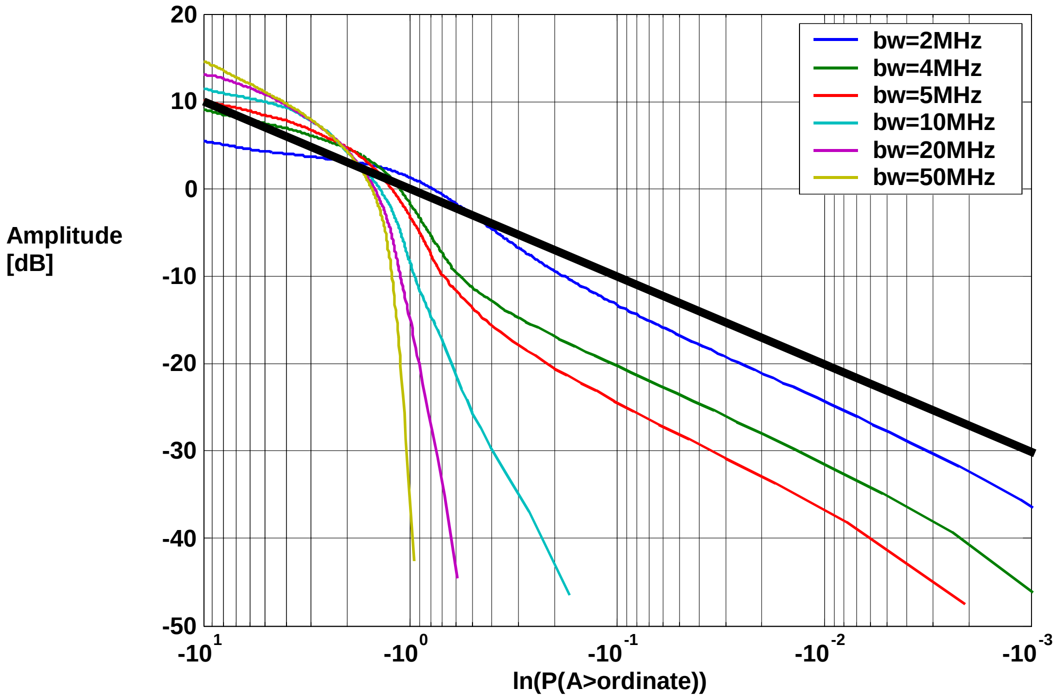


**Figure 6: Simulated APD plots for MB-OFDM measured in a variable bandwidth**

So far we have observed that the 3-band OFDM radio deviates from the ideal represented by AWGN. However, the Commission's Part 15 rules provide for a variety of UWB signaling schemes. The waveforms studied during the rulemaking proceeding included AWGN, direct sequence spread spectrum, and impulse. Flexibility exists to choose from a variety of modulation schemes according to market requirements. Therefore, it is very important to determine how the APD curve for our proposed waveform compares with that of other waveforms that were anticipated and allowed for UWB devices under Part 15.

To evaluate the MB-OFDM APD with allowed waveforms, we consider an impulse radio that uses bi-phase modulation and no dithering. In Figure 7, the impulse radio has a pulse repetition frequency of 1 MHz, while the victim receiver is chosen to have various bandwidths from 2 to 50 MHz.

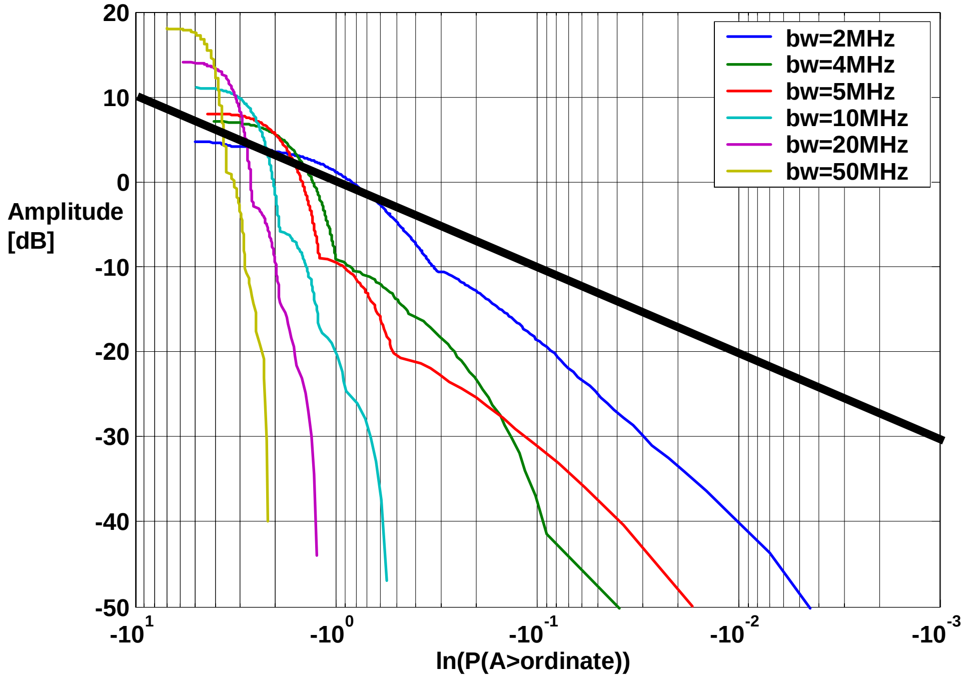


**Figure 7: Simulated APD plots for a 1 MHz BPSK impulse radio measured in a variable bandwidth**

### Peak Power Levels as a Function of Measurement Bandwidth

Figure 8 shows peak power values obtained from a simulation of a 1 MHz PRF impulse radio as well as the proposed MB-OFDM waveform. For reference the FCC peak-power limit is included as well as a continuous MB-OFDM waveform. It is shown that the MB-OFDM waveform behaves in a way similar to an impulse radio for low victim receiver bandwidths (having a peak power proportional to 20log$_{10}$(Bandwidth)), but later moderates to having a peak power proportional to only 10log$_{10}$(Bandwidth)). Thus, for wider bandwidths, <u>an advantage is seen in reduced peak power compared to the impulse radio</u>, which consistently obeys the 20log$_{10}$(Bandwidth)) as anticipated by the Commission's rules for peak power measurement.

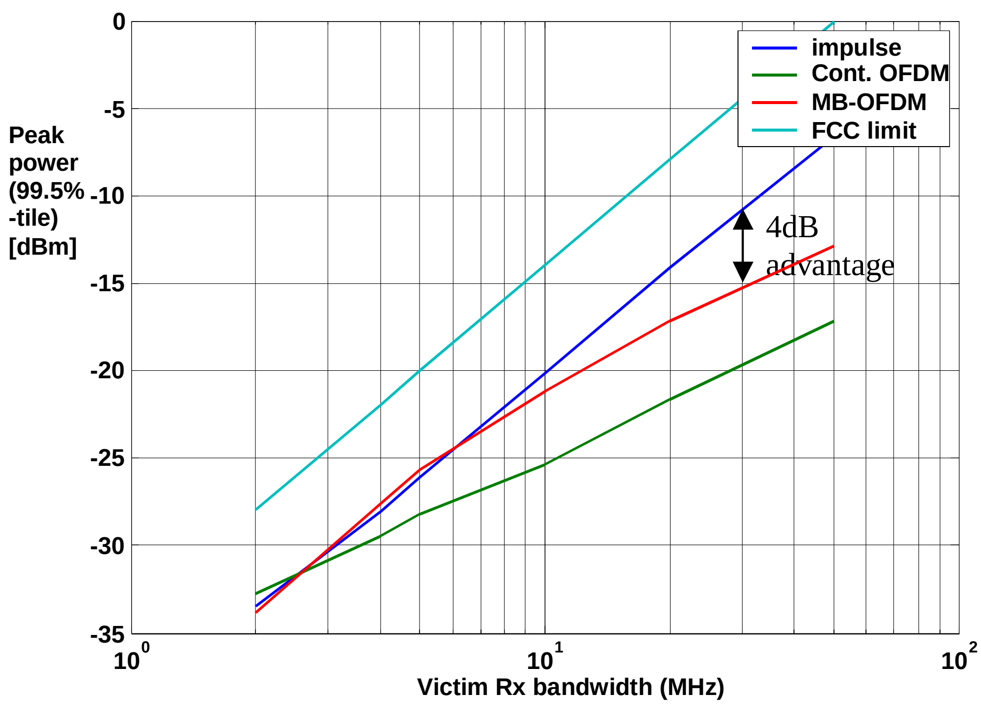


**Figure 8: Peak powers obtain for MB-OFDM and 1 MHz impulse radios**

## APD of MB-OFDM Compared to Impulse Radios

Figure 9 is included as a final comparison of APD plots for various candidate UWB waveforms. In this simulation experiment, the measuring filter was chosen to have a 50 MHz bandwidth and a roll-off factor of 0.5, and the pulse repetition frequency was varied from 1 to 10 MHz.

It is observed that the 10 MHz PRF impulse radio has a nearly identical APD to the MB-OFDM signal in the region of interest[6]. Furthermore, the 3 MHz and 1 MHz PRF radios have significantly higher required SIR ratios corresponding to the 1.8% P(A>ord.) line than the MB-OFDM system.

All the impulse radios studied comply with the rules in Part 15.

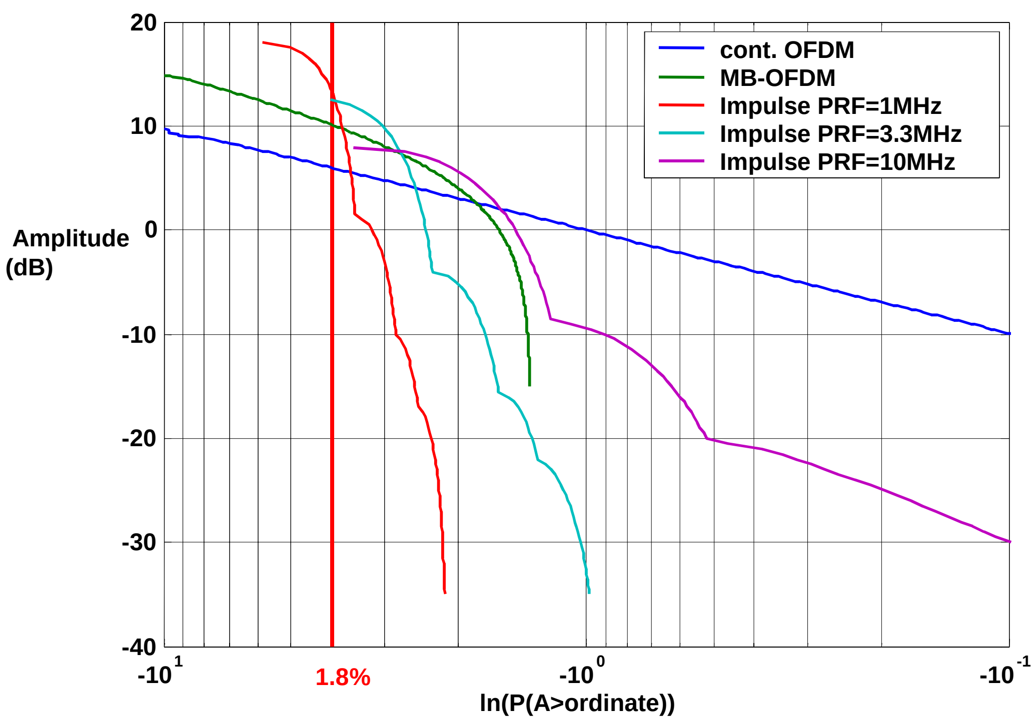


**Figure 9: APDs of OFDM signals and BPSK Impulse Radios**

[6] The "region of interest" is any part of the APD curve that exceeds the amplitude expected for complex Gaussian noise with non-negligible probability.

The bandwidth chosen for these simulations is 50 MHz, but the results can be generalized to other bandwidths provided the victim receiver bandwidth exceeds the UWB interferer PRF by a factor of 5 or more. In general, the ratio of PRF to victim receiver bandwidth determines the APD statistics as observed by a given victim receiver when other parameters are held constant.

## Other Mitigating Factors

The above analysis assumes that the dominant source of noise/interference is a *single instance* of the considered waveform. For this to be true, a single interferer must be very close to the victim receiver (within 10m to avoid thermal noise being a significant component of the N+I at the receiver). Furthermore, the system must have sufficient link margin to allow room for the interferer to overwhelm the receiver thermal noise floor and yet remain in the useful operating region. In cases when the link margin is small (such as Direct-to-Home (DTH) Satellite Television links), the C/I region of interest where the link starts to fail will always contain a significant component of thermal noise, which will tend to reduce the impact of any deviation from the ideal Rayleigh APD (see Figure 4 and Figure 5).

The following section illustrates a case where a single interferer is not the sole, dominant source of link degradation.

## Several Uncoordinated MB-OFDM Interferers

The additive combination of several uncoordinated UWB interferers combines to approximate a Rayleigh APD. This can be expected since the in-phase and quadrature components will tend to approximate a Gaussian distribution due to the Central Limit Theorem.

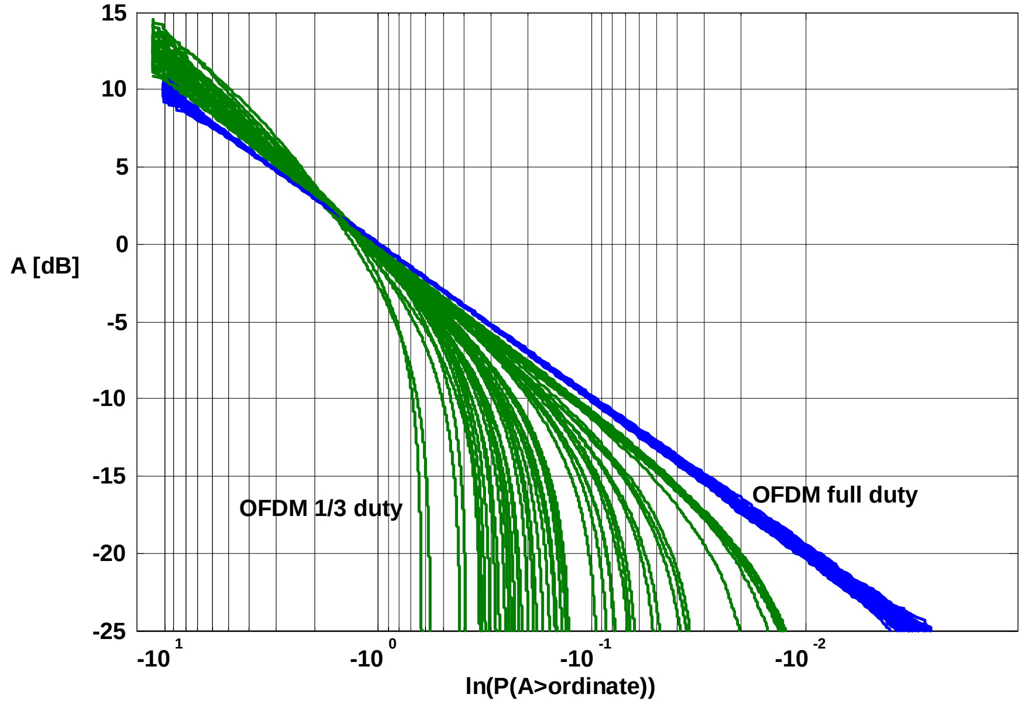


**Figure 10: Summation of 5 MB-OFDM Signals with randomly chosen delays**

As can be seen, the summation of only 5 uncoordinated interferers more closely approximates the ideal Rayleigh APD than a single MB-OFDM transmitter. In cases where a dense cluster of MB-OFDM devices is the source of interference observed from some distance, we can expect the approximation to the Rayleigh APD to be almost perfect, *i.e.*, the overall impact will be the same as Gaussian noise for any given interferer power.

## Conclusions

The APD analysis for the worst-case scenario of a single dominant interferer reveals the following:

- The required SIRs to obtain equivalent error protection for the 3-band OFDM waveform is lower than those needed for low PRF impulse radios for cases where the victim receiver band exceeds the impulse PRF by a factor of 5 (or more).

- The APD plots for lower bandwidth victim receivers show that peaks of the MB-OFDM signal are significantly attenuated by the Rx filter, bringing them closer to the ideal Rayleigh APD.

- The peak interference powers due to MB-OFDM are similar to those caused by a 1 MHz PRF impulse radio for <10 MHz victim receiver bandwidths, whereas for >10 MHz receiver bandwidths significantly lower peak powers are obtained for MB-OFDM than for the reference impulse radio.

In addition, there are two additional mitigating factors:

- Interference caused by a population of MB-OFDM devices will have a more benign aggregate APD than a single dominant interfering device.

- Receiver thermal noise will have a mitigating effect on the APD of an interfering MB-OFDM signal in many practical situations. In most practical scenarios this is expected to be a significant factor.

## PART 2: MB-OFDM INTERFERENCE TO QPSK TRANSMISSIONS

### Background

In part 1 APD plots were examined to understand the intrinsic interference properties of the MB-OFDM waveform. To provide reliable insight into the impact to a real transmission system, below we examine Uncoded QPSK transmissions of circa 33 MHz bandwidth (66 Mbps). This is a specific system that is somewhat representative of a digital satellite transmission system.

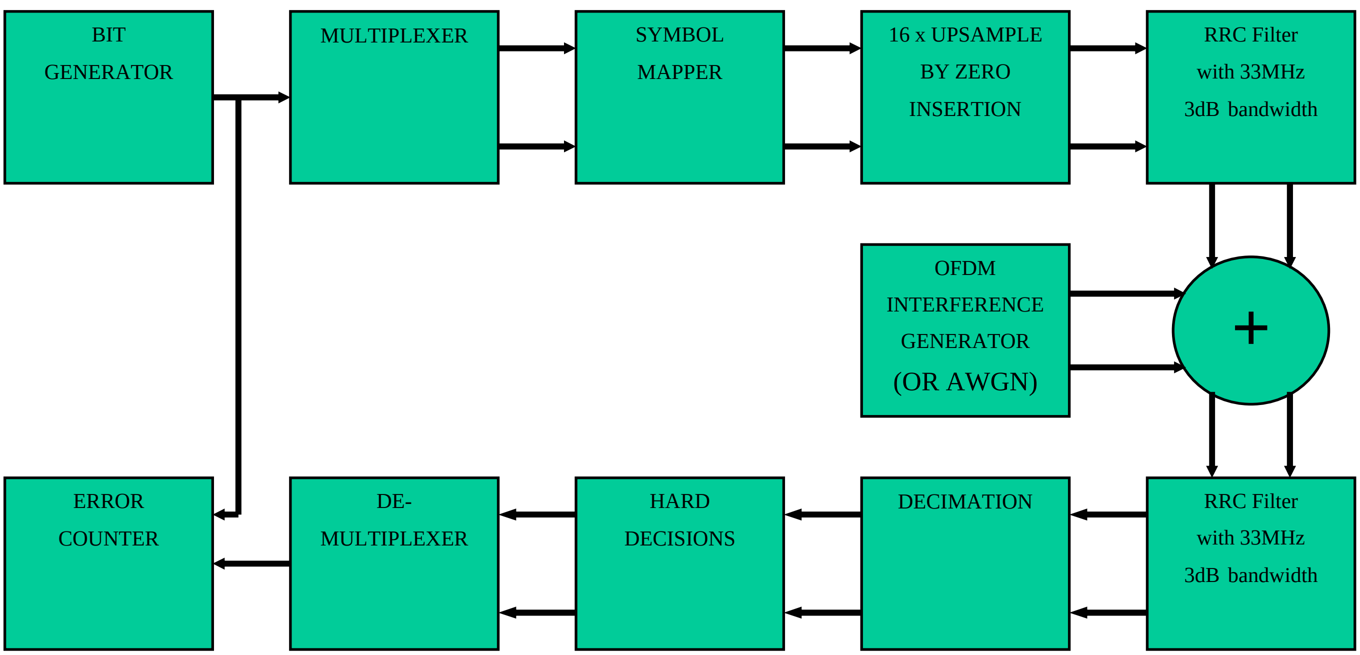


**Figure 11: Reference QPSK transmission system for interference study**

Figure 12 shows the reference scenario in the frequency domain. The interfering MB-OFDM sub-carriers are captured by a 33 MHz bandpass filter and 8 such sub-carriers are fully within the passband of the victim receiver. In all, 128 sub-carriers are simulated.

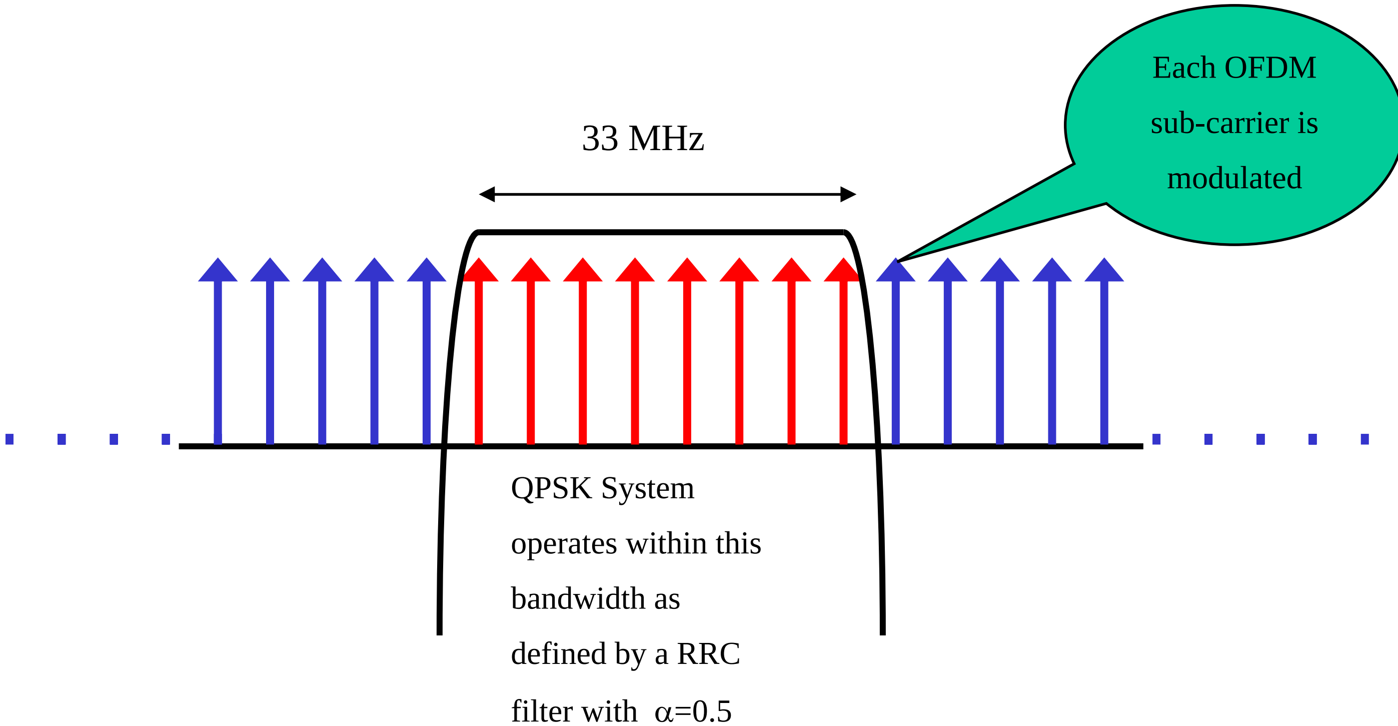


**Figure 12: Illustration of the simulation scenario in the frequency domain.**

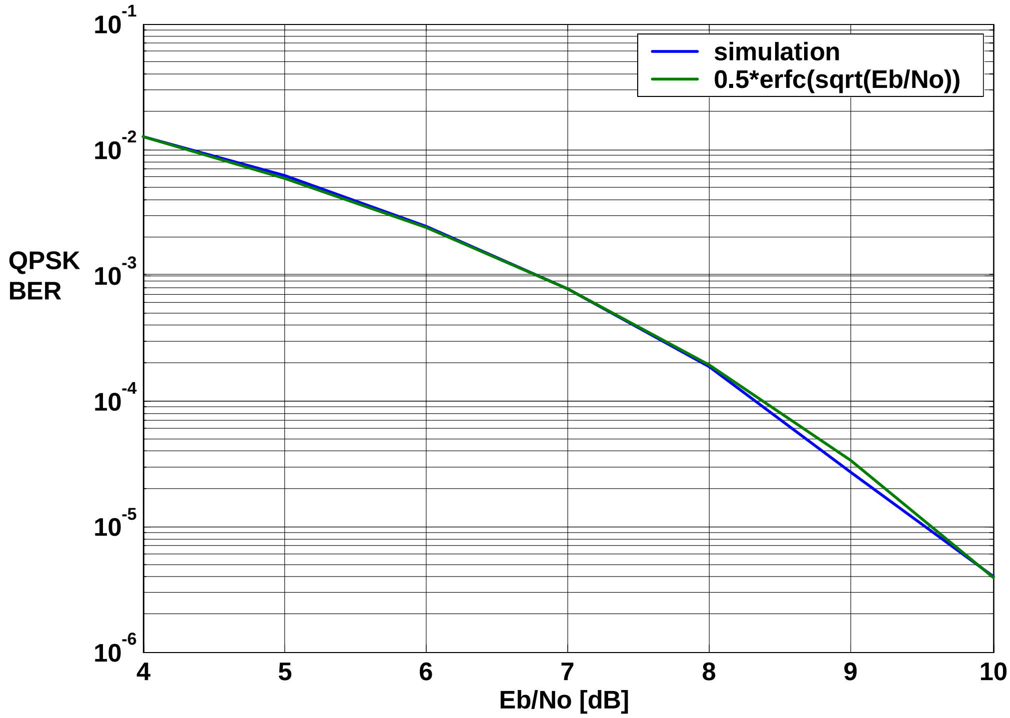


**Figure 13: BER obtained by simulation compared to theoretical expression**

Figure 13 is the BER plot obtained from the reference simulation. A comparison with the theoretical expression for the BER of an uncoded QPSK system is shown as the green trace on the plot. For comparison, the AWGN noise was replaced by a continuous OFDM waveform of identical power using 128 QPSK modulated tones. As mentioned previously, only 8 sub-carriers are fully within the passband of the victim receiver. The BER is slightly *less* than with a true Gaussian noise source because the tails of the distribution from the filtered OFDM signal are somewhat truncated compared to a true normal distribution. It should be noted that the area under the tails of each respective distribution directly corresponds to the bit error probability.

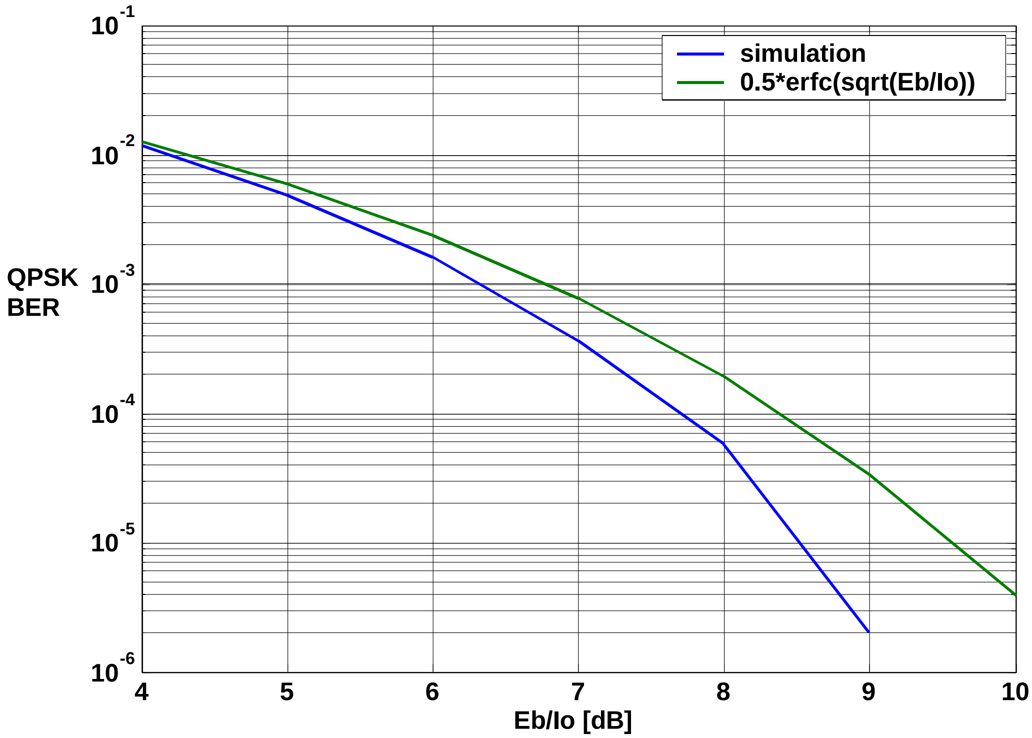


**Figure 14: BER obtained by simulation compared to theoretical expression**

## BER Prediction For $1/d$ Duty Cycle Noise Bursts As Interference

Given the analytical expression for BER of a QPSK modulation scheme, *i.e.*,

$$BER = 0.5\mathrm{erfc}\left(\sqrt{E_b/N_0}\right),$$

it can easily be shown that when the presence of the noise-like interference is governed by a regular duty cycle, the BER equation is modified as follows:

$$BER' = \left(0.5/d\right)\cdot\mathrm{erfc}\left(\sqrt{1/d}\,\sqrt{E_b/I_0}\right).$$

Following similar reasoning to the APD plots, we consider that the case thermal noise and background interference is negligible to be a special case of academic interest only. Thus, we introduce a constant background thermal noise density $N_0$ and consider the impact of an interference source of average spectral density $I_0$, which is subject to on/off keying according to a regular duty cycle factor $d$. We therefore obtain:

$\mathrm{BER} = 0.5 \cdot \mathrm{erfc}\left(\sqrt{E_b/(N_0 + d \cdot I_0)}\right)$ when the OFDM/noise burst is keyed "on" and

$\mathrm{BER} = 0.5 \cdot \mathrm{erfc}\left(\sqrt{E_b/N_0}\right)$ at all other times. Combining these, we obtain an average BER as:

$$\overline{\mathrm{BER}} = 0.5\left(\mathrm{erfc}\left(\sqrt{E_b/(N_0 + d \cdot I_0)}\right)/\, d + (d-1)\mathrm{erfc}\left(\sqrt{E_b/N_0}\right)/\, d\right)$$

For convenience in the simulations, we choose $d = 4$, which is slightly more pessimistic than the real case for MB-OFDM.

The QPSK victim receiver has a constant Eb/No of 10 dB (the uncoded BER is expected to be $BER = erfc\left(\sqrt{10}\right) \approx 3.87 \times 10^{-6}$). This situation is conservatively chosen to represent a healthy link margin for a typical QPSK transmission system employing typical forward error correction technology.

The simulation curve is obtained by introducing ¼ duty cycle MB-OFDM as an interferer, starting with Io=0 Watts and thus varying Eb/(No+Io) between the baseline Eb/No value of 10 dB and the final value of Eb/(No+Io)=3dB.

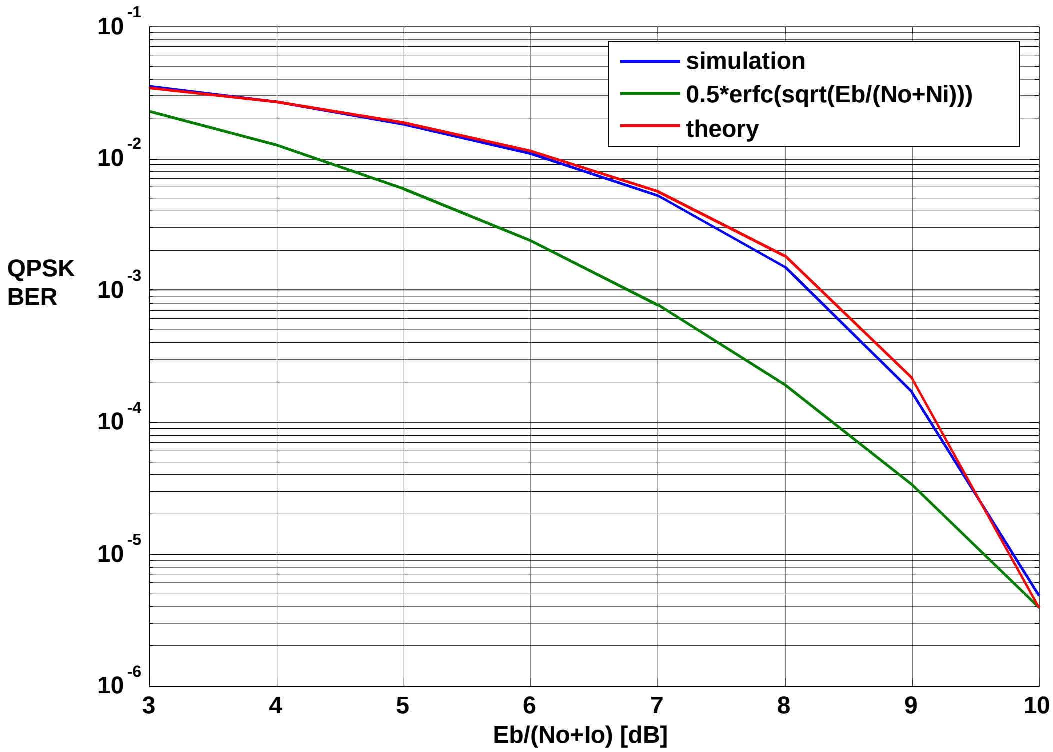


**Figure 15: Simulation With ¼ Duty OFDM Bursts As Interferer And Continuous Thermal Noise**

Figure 15 shows the results. Good agreement exists between the theoretical equation for BER (red curve) and the Monte-Carlo simulation (blue curve). The maximum Eb/No deviation between the theoretical AWGN BER (green curve) and the combined noise+interference case (blue curve) is limited to about 2 dB.

## Conclusions

- A continuous OFDM interferer has a more benign error inducing property than AWGN when each is applied at the same S/(I+N).
- Realistic conditions call for a non-zero value for background thermal noise for comparison of BER curves.
- In a reasonable test case (with $E_b/N_0 = 10\text{ dB}$), deviation of the BER curve from the AWGN case is limited to 2 dB.

**CONCLUSION**

Philips strongly endorses grant of the Petition. MB-OFDM employing a sequence of 3 bands is shown to create no more potential for interference than the impulse transmitters anticipated by the UWB rules. APD analysis shows that UWB impulse radios certifiable under Part 15 may require higher SIR ratios than the proposed MB-OFDM waveform for equivalent protection of a wideband receiver. Additional data demonstrate that MB-OFDM has peak power equivalent to that of an impulse radio of equivalent PRF when measured in low bandwidth receivers, and significantly lower peak power when measured in high bandwidth receivers. Favorable results also were obtained for simulations of QPSK transmission systems using a conservative link margin.

More specifically, from our analysis set forth above, we make the following conclusions:

- 3-band MB-OFDM showed a less harmful APD plot than impulse radios for all cases in which (Rx Bandwidth)/PRF > 5.
- Simulation of narrow bandwidth cases (<5 MHz) reveals a close resemblance of the APDs to impulse radios of the same PRF, and substantially lower peak-to-mean ratios compared to the wide bandwidth cases.
- Testing the impact of MB-OFDM on a QPSK transmission system showed that the required SNR increase always is less than 10log(*d*) (*e.g.*, for the a zero noise case), but the impact was reduced to below 2 dB in realistic scenarios with continuous AWGN also present.

For these reasons, the Commission should grant the waiver requested.

Respectfully submitted,

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September 29, 2004

## APPENDIX: Analytic Expression for APD of OFDM and MB-OFDM waveforms

For measurement bandwidths that exceed 20 MHz, the OFDM waveform has an approximately zero-mean, Gaussian PDF for the real and imaginary parts, each having a variance $\sigma^2$. Hence, the envelope, $r$, is approximately Rayleigh distributed[7].

Thus the PDF of the envelope is given by:

$$PDF(r) = \frac{r}{\sigma^2}\exp(-r^2/2\sigma^2), \quad r \geq 0$$

and the CDF can be readily evaluated as:

$$\begin{aligned} CDF(r) &= \int_0^r \frac{u}{\sigma^2}\exp(-u^2/2\sigma^2)\, du \\ &= 1-\exp(-r^2/2\sigma^2), \quad r \geq 0 \end{aligned}$$

Thus the CCDF is given as

$$1 - CDF = \exp(-r^2/2\sigma^2)$$

For unit power $2\sigma^2 = 1$. Therefore $APD = 1 - CDF = \exp(-r^2)$

Since $r^2 = 10^{A_{dB}/10}$, then $\log_{10}(-\ln(1-CDF)) = A_{dB}/10$, which explains the straight lines obtained for the Rayleigh distributed amplitudes shown throughout this submission.

We now introduce an arbitrary duty cycle *d*, to describe the impact of a MB-OFDM as experienced by a victim receiver operating in one of the visited bands.

During the active part of the duty cycle, the MB-OFDM interference is present with a variance given by $2d\sigma_i^2$ and due to the continuous presence of the Gaussian noise power, the total (N+I) variance is therefore $2d\sigma_i^2 + 2\sigma_n^2$. At all other times the noise variance is simply $2\sigma_n^2$. Thus we can write:

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[7] See for example, John G. Proakis, "Digital Communications", Third Edition, 1995, section 2-1-4 .

$$CDF = \frac{d-1}{d}\left[1-\exp\left(\frac{-r^2}{2\sigma_n^2}\right)\right] + \frac{1}{d}\left[1-\exp\left(\frac{-r^2}{2d\sigma_i^2 + 2\sigma_n^2}\right)\right]$$

$$= 1 - \left(\frac{d-1}{d}\right)\exp\left(\frac{-r^2}{2\sigma_n^2}\right) - \frac{1}{d}\exp\left(\frac{-r^2}{2d\sigma_i^2 + 2\sigma_n^2}\right)$$

so that,

$$CCDF = \frac{d-1}{d}\exp\left(\frac{-r^2}{2\sigma_n^2}\right) + \frac{1}{d}\exp\left(\frac{-r^2}{2d\sigma_i^2 + 2\sigma_n^2}\right)$$

This is the equation that was used to generate Figure 4: Analytic APD plots for MB-OFDM in a wide bandwidth. To fulfill the usual convention for APD plots of unit mean power, we apply the constraint $2\left(\sigma_n^2 + \sigma_i^2\right) = 1$, and set the ratio $\sigma_i^2 / \sigma_n^2$ according to the desired I/N ratio.

If we are interested in the noiseless case, this can be obtained by finding the limit of the above expression as $\sigma_n^2 \to 0$.

$$CCDF = \frac{1}{d}\exp\left(\frac{-r^2}{2d\sigma_i^2}\right)$$

and applying the constraint $2\sigma_n^2 = 1$, this reduces to

$$CCDF = \frac{1}{d}\exp(-r^2/d)$$

The curves presented in the body of this document (with the exception of Figure 4) have been derived from simulation experiments rather than analytically. However, the simple closed form expressions derived above may be used to quickly plot the expected APDs of OFDM waveforms with differing duty cycles and differing I/N ratios in order to obtain a reference comparison for experimental data.